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Precision Orbit Derived Total Density

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Atmospheric density modeling is the greatest uncertainty in the dynamics of low-Earth-orbit satellites. Accurate density calculations are required to provide meaningful estimates of the atmospheric drag perturbing satellite motion. This paper uses precision satellite orbits from the Challenging Mini Satellite Payload (CHAMP) as measurements in an optimal orbit determination process. The accuracy of the precision orbit derived density is compared to CHAMP accelerometer derived density for various filter/smoother settings to calibrate the precision orbit derived densities. Settings studied include the half-lives of the ballistic coefficient and density estimate Gauss–Markov processes, solution time spans, and baseline density models. The precision orbit derived densities are shown to more closely match the accelerometer derived densities along the CHAMP orbit than either the Jacchia 1971 empirical model or the High Accuracy Satellite Drag Model.

I. Introduction

OPERATIONAL orbit determination and orbit prediction experience suggests that the extreme upper atmosphere is more variable than the factors included in atmospheric density models commonly used in orbit propagation. Unmodeled atmospheric density variations can greatly impact the orbit determination process and add kilometers of error to orbit predictions.

The motivation for this research is to improve orbit determination and prediction by improving density models and to better measure and understand thermospheric and exospheric density variations, especially variations with time scales shorter than those of the empirical models typically used for atmospheric drag calculations. This paper represents a first step toward those long-term goals, which will process precision orbit data from multiple satellites simultaneously. Atmospheric density modeling has long been one of the greatest uncertainties in the dynamics of low-Earth-orbit satellites. Accurate density calculations are required to provide meaningful estimates of the atmospheric drag perturbing satellite motion. These effects increase with lower altitude orbits, higher effective area, and lower mass satellites.

McLaughlin [1] gave an introduction to the neutral atmosphere and the time varying effects on the density of the thermosphere and exosphere. The effects include diurnal variations, solar rotation, the solar cycle, winds and tides, gravity waves, long-term climate change, and magnetic storms and substorms. The authors hope to increase understanding of the variations in neutral density that are important for satellite drag. Vallado [2] gave an introduction to the basic density variations and to the density models most commonly used in orbit determination. A more comprehensive introduction to the space environment and the neutral atmosphere can be found in Hargreaves [3]. Sabol and Luu [4] gave a summary of the drivers of atmospheric density variations and some of the problems associated with the temporal resolution of various proxies used in empirical density models.

Marcos et al. [5] presented an overview of ongoing research to address the inaccuracies in satellite drag modeling. There are two major types of research ongoing to address these problems. The first is dynamic calibration of the atmosphere (DCA) and the second is use of satellites with accelerometers to measure the nonconservative accelerations, including drag. DCA involves estimating density corrections to a given atmospheric density model based upon the observed motion of satellites. The work of Storz et al. [6] and Bowman et al. [7,8] on the High Accuracy Satellite Drag Model (HASDM), Cefola et al. [9], Yurasov et al. [10,11], and Wilkins et al. [12,13] are all examples of approaches to provide corrections to atmospheric density models. In each case, the observations from a group of satellites are used to estimate large-scale corrections to an existing atmospheric density model. The approaches have shown the ability to provide a general improvement to a baseline atmospheric density model. The DCA approaches have several disadvantages, however. First, the approaches are designed to run internal to a particular orbit determination scheme. This means that users of other orbit determination schemes have to rely on that system to provide atmospheric density correction updates. In addition, the atmospheric density corrections are only applicable to a certain point in time. Thus, one must have access to the entire archive of density corrections applicable to a given problem. A second limitation of DCA approaches to date is that the corrections have limited spatial and temporal resolution. The corrections do allow the models to better represent effects with temporal resolution of several hours to days, but not temporal effects with shorter time scales. Most dynamic atmospheric density models use a daily solar flux and averaged 3 h geomagnetic indices as input values to address solar and geomagnetic activity. Using these values limits the ability of the models to represent changes in the atmosphere that occur within the averaging interval of the input data. Although the original Russian DCA work used radar observations, most current DCA approaches use two-line element sets of a large number of low-Earth-orbit objects as observations to develop atmospheric corrections in a DCA scheme. Unfortunately, relying on two-line element sets reduces the accuracy of the corrections and provides limited temporal resolution. HASDM [6–8] relies on the actual radar observations of low-Earth-orbit satellites, but even this accuracy is lower than that available from precision orbit ephemerides (POEs) or satellite laser ranging (SLR). In addition, the radar observations are not generally available.

The second major type of research related to improving atmospheric density knowledge is using satellites with accelerometers to measure nonconservative forces, which can then be used to estimate density. This represents the opposite extreme from using two-line element sets in terms of accuracy and total data availability. The accelerometer data provide a way to separate the gravitational forces from the nonconservative forces such as drag, solar radiation pressure, and Earth radiation pressure. Then by using accurate radiation pressure models, the drag acceleration can be determined

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very accurately with precise temporal resolution. The accelerometer measurements are extremely precise, but are currently only available for the Challenging Mini-Satellite Payload (CHAMP) and the two Gravity Recovery and Climate Experiment (GRACE) satellites. However, other satellites have flown with accelerometers in the past. For example, Rhoden et al. [14] used accelerometer data from the Satellite Electrostatic Triaxial Accelerometer experiment, which was flown at an altitude of around 200 km.

Some early results using the CHAMP accelerometer to estimate density were published by Konig and Neumayer [15] and by Bruinsma and Biancale [16]. Bruinsma et al. [17,18], and Nerem et al. [19] used CHAMP accelerometer data to derive atmospheric density. Schlegel et al. [20] used CHAMP accelerometer data to examine density structures over polar regions. In addition, several papers involving researchers at the University of Colorado and the French space agency, Centre National d'Études Spatiales, have used CHAMP and GRACE data to examine density variations during solar and geomagnetic events ([21–26]). Finally, Tapley et al. [27] described a technique for estimating density using the GRACE accelerometers. The CHAMP and GRACE data have great potential to add to the knowledge of the upper atmosphere, but unfortunately three satellites provide only limited spatial coverage at a given time, and that only for low altitudes.

The research presented in this paper represents a proof of concept of a way to combine the advantages of accurate data and good spatial coverage from many satellites. There are many satellites now carrying Global Positioning System (GPS) receivers or laser retroreflectors for SLR that, in combination with optimal orbit determination techniques, provide position accuracies in the range of a few centimeters to a few tens of centimeters. The precision GPS and SLR data (less than 10 cm precision) such as CHAMP will provide better accuracy than less precise GPS receivers, but the technique should work with lower density precision even for less accurate observations. This research uses the POE data available for these satellites as observations in an optimal orbit determination scheme that estimates density. In particular, this paper presents some results from estimating density using the CHAMP POEs and examines the calibration of the density estimation process. Using POE data provides fewer satellites than two-line element sets, but greatly increased accuracy of data that are nearly continuous. Conversely, POE data provide more satellites and decreased accuracy compared to accelerometer data. Several papers have looked at estimating nonconservative accelerations using GPS receiver or SLR observations. Doornbos et al. [28] looked at a type of differential correction using two-line element sets in a traditional DCA scheme and using a limited number of satellites with precision orbit information. A series of papers by van den IJssel and Visser ([29–31]) examined using GPS receiver data to estimate nonconservative accelerations as empirical accelerations in a scheme they named GPS accelerometry. They were able to reasonably reconstruct the in-track and cross-track accelerations derived from the CHAMP accelerometer, but only those with temporal resolutions of 20 min or more. Montenbruck et al. [32] examined the reconstruction of empirical accelerations for the GRACE-B satellite using both a batch and Kalman filter estimation technique. They showed that the overall variations in the empirical accelerations were similar between the two techniques, but a multiplicative bias existed between the acceleration magnitudes found from the two techniques. Finally, McLaughlin and Bieber [33,34] presented initial studies related to the current effort. McLaughlin and Bieber [33] examined the consistency of the CHAMP POE derived densities by comparing densities estimated as corrections to several models for the same time period and comparing densities estimated in orbit overlap solutions. Both comparisons gave consistencies with a worst case of about 10% of the estimated density. McLaughlin and Bieber [34] compared the POE derived densities to those derived from the accelerometer and found similar error ranges. The current research demonstrates significant improvement over the initial results and examines the effects of some of the parameters and models used in the density estimation.

II. Methodology

The results in this paper were created by taking CHAMP POE data and processing the positions and velocities as observations in an optimal orbit determination process to estimate density and ballistic coefficient. The POEs are available as either precision science orbits (PSOs) or as rapid science orbits (RSOs). Information about the processing and accuracy of the RSOs is available in several papers (Konig et al. [35,36] and Michalak et al. [37]). The accuracy of the RSOs is around 5–10 cm when compared to SLR. The PSOs are also mentioned in the above papers and were found as part of the gravity field solutions produced from CHAMP data. Published accuracies of the PSOs are not available, but presumably they are at least as accurate as the RSOs and probably slightly better. The PSO data are used when available, but there are no published PSOs available after 2003. This technique will work with less accurate data, but the corrections will be more dependent on the underlying density model.

The technique for estimating density in this paper is optimal orbit determination. The estimates will be optimal in the least-squares or minimum variance sense. POE data are used as measurements in a sequential measurement processing and filtering scheme. The filter estimates a state vector, including the position, velocity, atmospheric density, ballistic coefficient, and some other parameters, including those related to solar radiation pressure. Smoothing is applied to the solution to increase the accuracy by taking into account all the data in a given solution. The filter/smoother is able to estimate time-variable density and ballistic coefficient and includes realistic covariance matrices based on the physics of the problem. The force models other than drag included are the 90×90 GRACE Gravity Model 2 (GGM02C), solar radiation pressure (SRP), Earth infrared and albedo radiation pressure, lunisolar point masses, general relativity, and solid Earth and ocean tides. SRP was modeled assuming CHAMP was a sphere with area of 6.5 m^2, which represents an averaged area. Earth infrared and albedo radiation pressure models assumed a constant area of 20 m^2, where the difference from the SRP area accounts for the larger area projected toward Earth for CHAMP's geometry and attitude. Perfect absorption was assumed as a baseline for the radiation pressure modeling, but the coefficients are estimated. According to Bruinsma et al. [18], SRP is about 2 orders of magnitude lower than drag for the CHAMP altitude, and so these simplified models should introduce minimal error. Process noise was only included for the gravity model. Since covariance information is not available for GGM02C, the process noise level was determined using Joint Gravity Model 2 (JGM2) error levels for low-Earth-orbit satellites. This is the default setting in Orbit Determination Tool Kit (ODTK), which was used for this research. This is a conservative process noise level for the gravity model, since the JGM2 errors are larger than the GGM02C errors. The POE observations were provided every 30 s and given a noise level of 10 cm.

The density is estimated using the techniques of Wright [38] and Wright and Woodburn [39]. These techniques are currently available in the ODTK software package and are described below. Wright [38] has developed a technique to estimate local atmospheric density in real-time as part of the orbit determination process. The technique relies on an optimal estimation scheme that will be described in more detail later. This is a significant advance over the normal technique of estimating the ballistic coefficient (BC) or drag coefficient because BC estimates absorb errors in modeling of both atmospheric density and BC, and often include geopotential errors as well. Wright and Woodburn [39] show atmospheric density and ballistic coefficient are simultaneously observable in a filtering scheme. They also show that using 3 h step functions of geomagnetic indices does not satisfy the McReynold's filter-smoother consistency tests, but using polynomial spline fits to the 3 h data does satisfy the test. This shows the need to have smooth data over shorter time scales than currently available. Tanygin and Wright [40] develop a polynomial spline fit technique for geomagnetic indices that is available in Wright's density estimation scheme. However, the splines were not used for this research.

The atmospheric density is estimated as a correction to an atmospheric model, and so the state parameter estimated is density

Table 1 Solar and geomagnetic indices

Date	3 h planetary equivalent amplitude indices, a_p	Daily planetary amplitude A_p	Observed 10.7 cm daily solar radio flux, F10.7
17 Feb. 2002	12 18 27 9 9 7 4 7	12	196.6
18 Feb. 2002	4 7 7 5 6 12 22 22	11	192.8
19 Feb. 2002	15 4 6 3 3 4 7 5	6	189.4
20 Feb. 2002	6 3 7 6 12 9 7 12	8	193.4
21 Feb. 2002	5 7 9 6 7 6 7 15	8	201.1
10 Mar. 2005	22 18 15 12 12 22 9 5	14	101.6
11 Mar. 2005	9 9 6 4 4 3 0 5	5	104.9
12 Mar. 2005	6 3 4 0 2 3 2 6	3	110.1
13 Mar. 2005	5 0 0 3 3 5 9 18	5	113.8
14 Mar. 2005	32 27 12 15 39 12 7 9	19	111.5
16 Mar. 2005	3 2 3 6 15 7 7 7	6	104.6
17 Mar. 2005	15 22 9 9 6 32 7 5	13	101.4
18 Mar. 2005	3 4 5 6 3 6 27 27	10	96.5
19 Mar. 2005	39 27 6 6 5 0 2 4	11	93.0
20 Mar. 2005	2 7 4 4 4 2 4 2	4	89.0

correction divided by the model density. The correction can be applied to the Jacchia 1971 [41], Jacchia–Roberts [42], CIRA 1972 [43], MSISE-1990 [44], or NRLMSISE-2000 [45] models. Since Jacchia 1971, Jacchia–Roberts, and CIRA 1972 were all derived from the work of Jacchia, the results from these three models should be similar. There are two corrections applied to the model. The first is a baseline correction that is derived from historical F10.7 (solar flux) and a_p (geomagnetic activity) measurements. This correction is for consistent errors in the baseline density models observed in historical orbit determination data that change with solar and geomagnetic activity. An exponential Gauss–Markov sequence propagates the baseline density correction at perigee height. A transformation relates atmospheric density error at perigee to that at the current point in the orbit. Since CHAMP is in a near-circular orbit, any errors caused by the transformation will be minimized.

The second atmospheric density correction is a dynamic correction based on current conditions and using the orbit measurements. Because this is a sequential process, a correction can be estimated at each time step in the filter instead of doing a single correction for the whole time span of data used in a batch least-squares process. The dynamic correction also has associated exponentially correlated Gauss–Markov processes describing the modeling errors. The half-life of the exponential Gauss–Markov process can be specified by the user for both the density and ballistic coefficient. This determines how much past data will affect the individual density correction at each time step. The ballistic coefficient was initialized using the yearly averages found by Bowman et al. [46] (0.00444 m^2/kg for 2002–2003 and 0.00436 m^2/kg for 2004–2005), but BC is estimated as part of the filter/smoother process. These BC values provided reasonable starting points for the estimation process.

The densities derived from the POE data are compared to those derived from the CHAMP accelerometer.[§] Bruinsma's method of data processing is described in [16–19]. The accelerometer density values are averaged to 10 s intervals. For moderate geomagnetic activity the precision of the accelerometer densities is a few percent; the accuracy is below 5% for latitudes below 60 deg and 5–10% for higher latitudes. For geomagnetic storm conditions the accuracy drops to 5–10% for latitudes below 60 deg and 15–40% for high latitudes. The largest error source is winds, especially at high latitudes and during geomagnetic storms, followed by errors in drag coefficient modeling. More specifics on the errors can be found in Bruinsma et al. [18].

POE derived density solutions were developed for 17–21 February 2002 and 10–20 March 2005. These represent time spans shortly after solar maximum and shortly before the solar minimum. Both time periods represent relatively quiet times in terms of solar and geomagnetic activity, as shown in Table 1. Both solar maximum and solar minimum times could prove more challenging for the density estimation, but the time periods used here provide an initial feasibility study of the density estimation technique employed in this paper.

Solutions for density were obtained on the above dates using all combinations of BC and density Gauss–Markov process half-lives of 1.8, 18, 180, 1800, 18,000, and 180,000 min for all available baseline density models. The February 2002 solutions were 36 h solutions using PSO data and the March 2005 solutions were 14 h using RSO data. Various solution time spans were also investigated. The results are given in the following section.

III. Results

For all cases examined, the residuals of the solutions compared to the precision ephemerides and the position and velocity filter-smoother consistency checks were examined. The filter often threw out a significant portion of the data for BC half-lives greater than 180 min. Since the data represent precision ephemerides, solutions for density with BC half-lives over 180 min are excluded from further consideration. Typical residuals for the solutions fall within 4 to 5 cm, which is comparable to the accuracy of the precision orbit data used as observations. The initial qualitative results examine solutions for 20–21 February 2002 and 17 March 2005 to show typical trends. Then composite quantitative results are tabulated at the end of the section.

A. Varying Ballistic Coefficient Half-Life

The effects of varying the half-life of the exponentially decaying Gauss–Markov process for ballistic coefficient are examined in this subsection. The density half-life was fixed at 1.8 min on 20–21 February 2002 and at 18 min on 17 March 2005. These density half-lives correspond to values giving the best density estimates compared to accelerometer derived density. The density estimates produced in this research are corrections to an existing density model. The baseline density model was Jacchia–Roberts for the 20–21 February 2002 and NRLMSISE-2000 for 17 March 2005. Jacchia–Roberts gave better results as the baseline density model than NRLMSISE-2000 for March 2005, but the NRLMSISE-2000 results were included to show a range of results. Different density half-lives and baseline models were used to show that the results are consistent, independent of those factors. In addition, the data presented here cover 12 h of the solution time span so that the differences can more easily be seen in the figures. However, the results given are representative of those throughout the solution time span, except for some minor effects at the beginning and end of the total time span.

Figure 1 shows the effects of BC half-life on density estimation and comparison to Bruinsma's (see footnote [§]) accelerometer derived density for 20 February 2002. The first noticeable feature is the

[§]Private communication, S. Bruinsma of Centre National d'Études Spatiales, 2008.

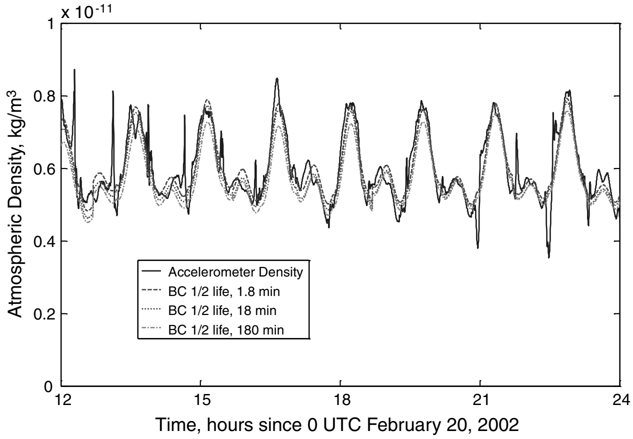

Fig. 1 Effects of ballistic coefficient half-life (20 February 2002). Density half-life is 1.8 min and Jacchia–Roberts is the baseline density model.

accelerometer is able to observe spikes in density that the POE method is not able to observe, but the POE derived densities track the major variations very well. The results show that as BC half-life is increased the density estimate is decreased. The BC half-lives of 1.8 and 18 min give the best match to the accelerometer derived density. However, the low half-lives do not correspond to temporal resolution as low as the half-lives. The temporal resolution is around 20–30 min, which matches previous work [29–31].

The original work of Wright [38] and Wright and Woodburn [39] had the BC and density half-lives significantly different to have simultaneous observability of BC and density. In particular, they used a long BC half-life. This contradicts the results found here. However, the original work used intermittent observations, while this work uses POEs based on continuous GPS. Therefore, the observability should be improved. Studies of BC variation using this technique for CHAMP have shown that there is very little correlation of the difference in accelerometer density and POE density with the estimated BC and that BC does not significantly diverge from the expected values. This implies that very little density error is being absorbed by the BC estimation, since a high negative correlation would be expected if density error was absorbed by the BC estimate. In addition, the fact that POE density matches accelerometer density so well shows that the density is observable in this scenario. The estimated ballistic coefficient has a high-frequency variation of about 0.5% during portions of the solutions using the 1.8 min BC half-life. This variation could correspond to small changes in attitude, which is the only factor in BC likely to change so quickly.

Figure 2 shows the effects of BC half-life on density estimation for 17 March 2005. The figure is similar to Fig. 1, except that there are no significant spikes in the accelerometer data. Another notable difference is that the density peaks on 17 March 2005 are comparable to the 20 February 2002 date, but the minimums are much lower. There are two offsetting factors affecting this. The first is that the solar cycle is much closer to minimum in 2005 and the second is CHAMP is at a lower altitude in 2005 than 2002. All POE derived densities are very similar and tend to overestimate the peaks compared to the accelerometer data and miss the low-magnitude peak observed in the accelerometer data. The POE density better matched the accelerometer density peaks when using Jacchia-based models. This shows the underlying dependence of the technique on the baseline density model. The smaller peaks are not observed in the POE data, which is related to the fact that the baseline density model does not model the peaks and the temporal resolution of the technique.

The density half-life was 1.8 min for the 20 February 2002 data presented and 18 min for 17 March 2005. The baseline model used was also changed from Jacchia–Roberts on 20 February 2002 to NRLMSISE-2000 on 17 March 2005. The trend of the density estimate decreasing as BC half-life increased and of shorter BC half-lives giving better solutions was consistent, but not universal, regardless of the density half-life used and of the baseline model from which the POE density estimates were derived.

B. Varying Density Coefficient Half-Life

The effects of varying the half-life of the exponentially decaying Gauss–Markov process for density are examined in this subsection. The BC half-life was fixed at 1.8 min on 20–21 February 2002 and 17 March 2005. These BC half-lives correspond to values giving the best density estimates compared to accelerometer derived density. The baseline density model was again Jacchia–Roberts for 20–21 February 2002 and NRLMSISE-2000 for 17 March 2005. The data presented again cover 12 h of the solutions, but the results given are representative of those throughout the solution time span. Figure 1 differs from Fig. 3 and Fig. 2 differs from Fig. 4 because Figs. 1 and 2 vary the BC half-life and Figs. 3 and 4 vary the density half-life.

Figure 3 presents the effects of varying density half-life for 20 February 2002. The trend is that the primary density peaks increase and the small density peaks decrease as the density half-life is increased. In terms of the major density peaks, this is the opposite trend exhibited by the BC half-life. The figure shows no clear distinction in terms of which density half-lives give the best solutions.

Figure 4 shows the effects of density half-life for 17 March 2005. The trend of increasing density estimate as density half-life is increased is not as consistent on this date. Once again, the density half-lives of 1.8 and 18 min give the solutions that best match the accelerometer density. Overall, density half-lives of 1.8 and 18 min

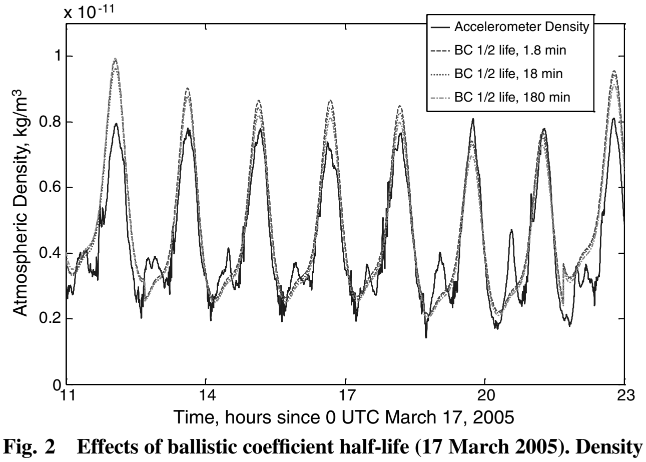

Fig. 2 Effects of ballistic coefficient half-life (17 March 2005). Density half-life is 18 min and NRLMSISE-2000 is the baseline density model.

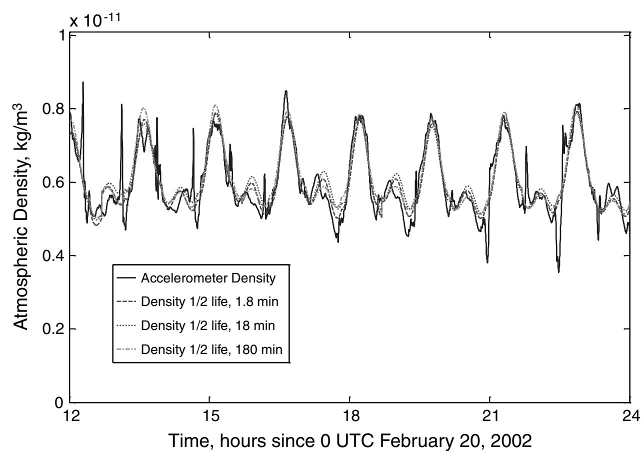

Fig. 3 Effects of density half-life (20 February 2002). Ballistic coefficient half-life is 1.8 min and Jacchia–Roberts is the baseline density model.

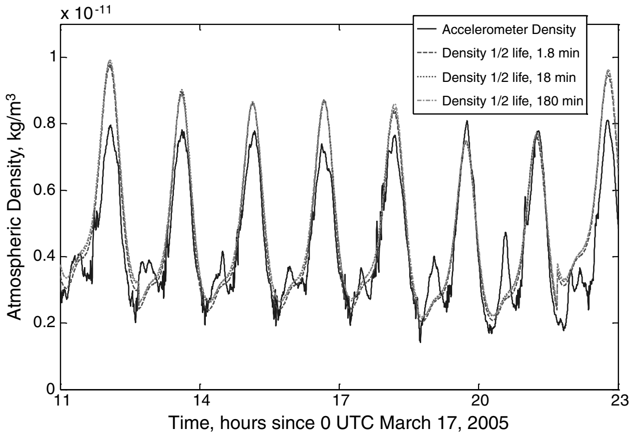

Fig. 4 Effects of density half-life (17 March 2005). Ballistic coefficient half-life is 1.8 min and NRLMSISE-2000 is the baseline density model.

gave superior solutions regardless of BC half-life or baseline density model. However, the differences in density estimates change little as density half-life is changed. The results using NRLMSISE-2000 as the baseline model significantly overestimate the peaks compared to the accelerometer, as shown in Figs. 2 and 4.

C. Varying Baseline Density Model

This subsection covers the effects of the underlying baseline density model. The BC half-life and density half-life were both fixed at 1.8 min on 20–21 February 2002, but the BC half-life was changed to 18 min on 17 March 2005. These half-lives correspond to values giving the best density estimates compared to accelerometer derived density, but the results are comparable for any combination of half-lives of 1.8 or 18 min. The data presented again cover 12 h of the solution time span. Solutions were found using Jacchia 1971, Jacchia–Roberts, CIRA 1972, MSISE-1990, and NRLMSISE-2000 as baseline densities models. However, Jacchia 1971, Jacchia–Roberts, and CIRA 1972 all gave results that were nearly identical, and so only Jacchia–Roberts results are presented.

Figure 5 presents the effects of varying baseline density model for 20 February 2002. The major result is that all the baseline models give comparable solutions for this time period, but the Jacchia-based models better match the accelerometer data. Ideally, the corrections would give very similar results for any baseline density model, since the correction is attempting to find the actual density. However, the nature of using a Gauss–Markov process (or any other method) to estimate a correction to a baseline model will mean that the density

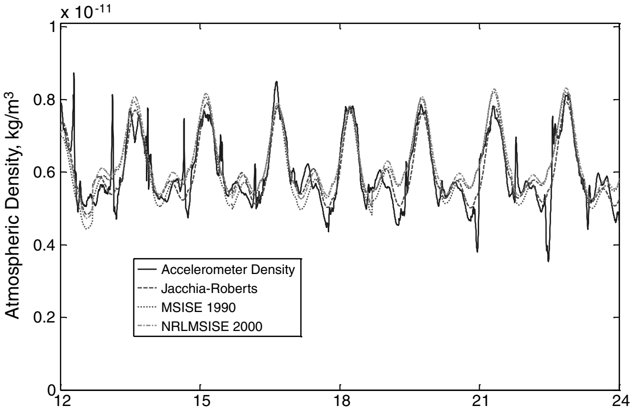

Fig. 5 Effects of baseline density model (20 February 2002). Ballistic coefficient half-life is 1.8 min and density coefficient half-life is 1.8 min. The densities shown are the POE derived densities from the baseline model and not the actual model densities.

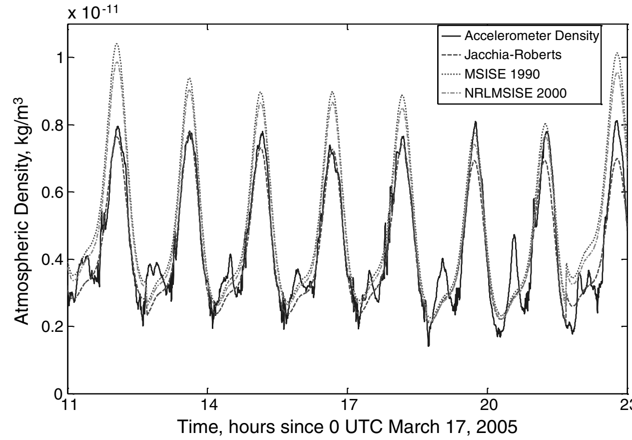

Fig. 6 Effects of baseline density model (17 March 2005). Ballistic coefficient half-life is 18 min and density coefficient half-life is 1.8 min. The densities shown are the POE derived densities from the baseline model and not the actual model densities.

correction will always be dependent on the baseline model. This is proven by the results in this section.

The effects of varying baseline density model for 17 March 2005 are shown in Fig. 6. The Jacchia-based models again better match the

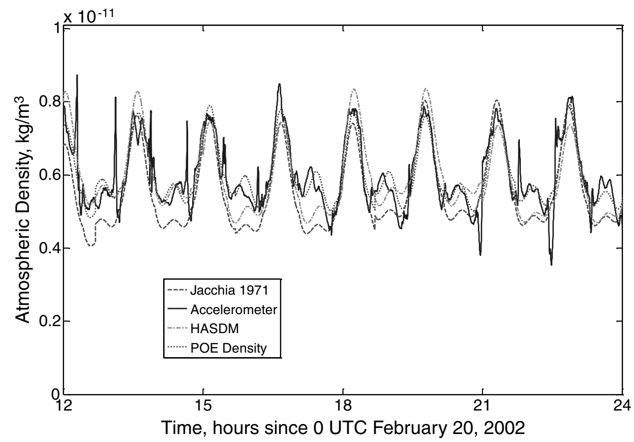

Fig. 7 Density comparison plot (20 February 2002). POE solution found with ballistic coefficient half-life of 1.8 min, density coefficient half-life of 1.8 min, and Jacchia–Roberts baseline model compared to Jacchia 71 model, HASDM density, and accelerometer density.

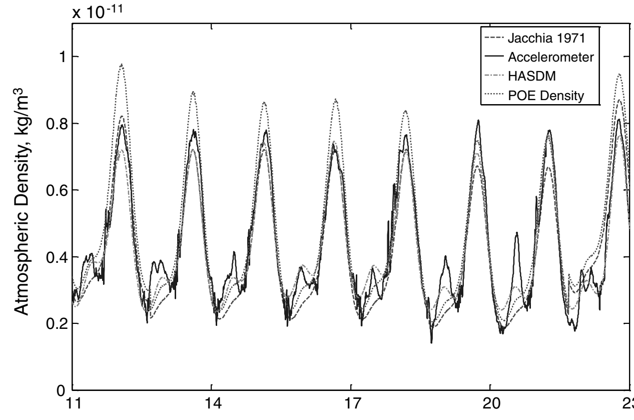

Fig. 8 Density comparison plot (17 March 2005). POE solution found with ballistic coefficient half-life of 1.8 min, density coefficient half-life of 1.8 min, NRLMSISE-2000 baseline model compared to Jacchia 71 model, HASDM density, and accelerometer density.

Table 2 Time-averaged cross-correlation of POE density with accelerometer density for various density and BC half-life combinations and various baseline density models (17–21 February 2002); the cross-correlation for uncorrected Jacchia 1971 is 0.827 and for HASDM is 0.846

Half-life (min): density, BC	CIRA 1972	Jacchia 1971	Jacchia–Roberts	MSISE 1990	NRLMSISE 2000
1.8, 1.8	0.899[ab]	0.898[a]	0.899[a]	0.835	0.855[a]
1.8, 18	0.894	0.893	0.894	0.832	0.848
1.8, 180	0.874	0.873	0.874	0.809	0.825
18, 1.8	0.896	0.895	0.896	0.801	0.828
18, 18	0.890	0.889	0.890	0.800	0.823
18, 180	0.849	0.848	0.849	0.760	0.780
180, 1.8	0.889	0.888	0.889	0.839[a]	0.854
180, 18	0.879	0.879	0.879	0.838	0.849
180, 180	0.805	0.806	0.805	0.771	0.781

[a]Indicates best correlation in column. [b]Indicates best overall correlation.

Table 3 Time-averaged cross-correlation of POE density with accelerometer density for various density and BC half-life combinations and various baseline density models (10–20 March 2005); the cross-correlation for uncorrected Jacchia 1971 is 0.950 and for HASDM is 0.971

Half-life (min): density, BC	CIRA 1972	Jacchia 1971	Jacchia–Roberts	MSISE 1990	NRLMSISE 2000
1.8, 1.8	0.968	0.968[a]	0.968[ab]	0.960[a]	0.960[a]
1.8, 18	0.964	0.964	0.964	0.957	0.958
1.8, 180	0.960	0.960	0.960	0.954	0.955
18, 1.8	0.967	0.967	0.967	0.957	0.958
18, 18	0.966	0.966	0.966	0.955	0.956
18, 180	0.957	0.957	0.957	0.948	0.949
180, 1.8	0.968[a]	0.968	0.968	0.957	0.958
180, 18	0.966	0.966	0.966	0.954	0.955
180, 180	0.958	0.959	0.959	0.949	0.950

[a]Indicates best correlation in column. [b]Indicates best overall correlation.

accelerometer data for this time period and the dependence of the technique on the baseline density model is more apparent on this day.

D. Varying Solution Time Span

Solution time spans with lengths of 36 h, 24 h starting every 12 h (2 total), 12 h starting every 6 h (5 total), and 6 h starting every 3 h (11 total) were examined for 20–21 February 2002. On 17 March 2005 the solution time span lengths examined were 24 h, 12 h starting every 6 h (3 total), and 6 h starting every 3 h (7 total). Density estimates from the shorter solution time spans matched the longer solution time spans nearly exactly, except for some slight end effects. The solution time span is not a significant factor in the accuracy of the density estimates over ranges of 6 to 36 h. However, the results near the beginning and end of the solution time span should always be used with care, especially for the shorter time spans.

E. Comparison of Various Density Techniques

The effects of varying parameters on the ability of the POE derived densities to match accelerometer derived densities were examined in previous subsections. However, the question still remains about the accuracy of the POE derived densities compared to other techniques. This section examines the Jacchia 1971 model densities, HASDM densities derived from DCA, the POE derived densities, and Bruinsma's (see footnote §) accelerometer derived densities. The HASDM densities were provided¶ in March 2009. The POE derived densities were found using BC and density half-lives of 1.8 min. The 20 February 2002 solutions used Jacchia–Roberts as the baseline density model and those on 17 March 2005 used NRLMSISE-2000.

Figure 7 shows the comparisons for 20 February 2002. The figure shows that, overall, the POE derived density clearly matches the accelerometer density better than the other two techniques and that HASDM outperforms the Jacchia 1971 model.

Figure 8 shows the comparison for 17 March 2005. The figure shows that HASDM best matches the accelerometer density. The POE density overestimates the major peaks and underestimates the minor peaks seen in the accelerometer density.

¶Private communication, B. Bowman, 2009.

F. Quantitative Analysis of Various Factors

This section uses the zero delay cross-correlation coefficient to quantitatively compare different estimated densities to the accelerometer derived density of Bruinsma (see footnote §). The cross-correlation was used instead of the root mean square (rms) because the density sources, including accelerometer density, almost certainly have biases that would skew the rms. However, the rms trends are very similar to those seen in the cross-correlations.

Table 2 shows the time-averaged cross-correlation using each of the available baseline density models and all the combinations of density and ballistic coefficient half-lives for all the February 2002 days examined. The correlations for the Jacchia 1971 empirical model and HASDM are also given. The table shows that the Jacchia-based models gave better results than the mass spectrometer and incoherent scatter (MSIS) models when used as the baseline model. The table also shows that lower values of the half-lives gave better results. Finally, the results show that the best cases for POE density performed better than the empirical model and HASDM.

Table 3 shows the same results for the March 2005 days. Note that the correlations are higher for 2005, which is much closer to solar minimum. The Jacchia-based models again outperform the MSIS models when used as the baseline model, and the lower values of half-lives give better results. However, in this case HASDM slightly better matches the accelerometer data than the POE density for this time period. The POE density is still considerably better than the empirical model.

Table 4 shows the combined results for both time periods examined in this study. The results confirm that overall the solutions with low half-lives and using the Jacchia-based models as baseline models provide the best POE density solutions compared to the accelerometer. In addition, the POE density is somewhat better than the HASDM density on average and both are significantly better than the empirical Jacchia 1971 model with no corrections. One

Table 4 Time-averaged cross-correlation of POE density with accelerometer density for various density and BC half-life combinations and various baseline density models (17–21 February 2002 and 10–20 March 2005); the cross-correlation for uncorrected Jacchia 1971 is 0.916 and for HASDM is 0.936

Half-life (min): density, BC	CIRA 1972	Jacchia 1971	Jacchia–Roberts	MSISE 1990	NRLMSISE 2000
1.8, 1.8	0.949[a]	0.948[a]	0.949[ab]	0.925[a]	0.931[a]
1.8, 18	0.945	0.945	0.945	0.922	0.927
1.8, 180	0.936	0.935	0.936	0.914	0.918
18, 1.8	0.947	0.947	0.947	0.913	0.922
18, 18	0.946	0.944	0.945	0.911	0.919
18, 180	0.927	0.926	0.927	0.895	0.902
180, 1.8	0.946	0.946	0.946	0.924	0.929
180, 18	0.941	0.942	0.942	0.921	0.925
180, 180	0.915	0.916	0.916	0.899	0.902

[a]Indicates best correlation in column. [b]Indicates best overall correlation.

significant note is that HASDM uses CHAMP as one of the satellites in the overall HASDM density solution. Therefore, the HASDM density along the CHAMP orbit should be better than the HASDM density along a satellite orbit not included in the HASDM solution.

IV. Conclusions

This paper has presented calibration studies for estimating total atmospheric density along the orbit of a satellite for which precision orbit ephemerides are available. Total density has been estimated along the CHAMP orbit by using POE data as measurements in an optimal orbit determination process. The study covered 17–21 February 2002 and 10–20 March 2005. The total density estimates were created using various values of half-life of the exponentially decaying Gauss–Markov processes for both density and ballistic coefficient estimation. In addition, corrections to several different baseline atmospheric density models were presented. The density estimates were compared to density derived from the CHAMP accelerometer. The accelerometer densities were considered truth for comparison purposes.

The effects of varying the BC half-life and density half-life were examined in detail. The results showed that BC and density half-lives of 1.8 and 18 min gave better results compared to much longer values for any value of density half-life and any baseline density model. In addition, for the cases where BC half-life was higher than 180 min, the filter in some cases threw out a significant number of data points even though those data points were precision ephemeris data. Further study of the effects of BC half-life variations in the neighborhood of 1.8 to 18 min is warranted in the future.

Examination of different baseline density models showed the Jacchia-based models consistently outperformed the MSIS-based models when used as a baseline density model. The differences in the estimated density when using different baseline models showed the dependence of the technique on the underlying baseline density model. This research also examined the effects of solution time spans of 6, 12, 24, and 36 h on the solution. Other than end effects, the solutions showed negligible change as a function of solution time span.

The POE derived densities were compared to those of the Jacchia 1971 empirical model, HASDM, and accelerometer. The POE derived densities (overall cross-correlation of 0.946) were clearly superior to Jacchia 1971 (overall cross-correlation of 0.916) in all cases when compared the accelerometer derived density. The POE derived density was superior to the HASDM density (cross-correlation of 0.899 vs 0.846) for February 2002, but slightly worse for March 2005 (cross-correlation of 0.968 vs 0.971). Overall the POE densities better matched the accelerometer variations than Jacchia 1971 or HASDM. However, the POE densities were not able to match the temporal resolution of the accelerometer density, since the accelerometer density contains many short-period spikes that do not appear in any of the other data sets. In addition, both the POE and HASDM densities are tied to the underlying empirical model, and so they cannot adequately track all the variations observed by the accelerometer.

This paper has presented a new technique to estimate density along the track of a satellite using precision orbit ephemerides. Future work in this area will look at many more time periods, especially those with higher levels of solar and geomagnetic activity and very low levels of solar activity to determine if the results found so far are independent of these factors. In addition, the ability of the POE method to observe short-period variations in density needs further investigation to determine the minimum temporal resolution of the technique and to relate variations to specific upper atmospheric phenomena. Finally, the work needs to be expanded to the GRACE satellites for further calibration of the technique before including satellites that have POE data, but no accelerometers.

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References

[1] McLaughlin, C. A., "Upper Atmospheric Phenomena and Satellite Drag," *Advances in the Astronautical Sciences*, Vol. 123, 2005, pp. 989–996; American Astronautical Society, Paper 05-315.

[2] Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, 3rd ed., Microcosm Press, El Segundo, CA, 2007, Chap. 8, App. A.

[3] Hargreaves, J. K., *The Solar-Terrestrial Environment*, Cambridge Univ. Press, Cambridge, MA, 1992.

[4] Sabol, C., and Luu, K. K., "Atmospheric Density Dynamics and the Motion of Satellites," *AMOS Technical Conference*, Wailea, HI, Sept. 2002.

[5] Marcos, F. O., Wise, J. O., Kendra, M. J., and Grossbard, N. J., "Satellite Drag Research: Past, Present, and Future," *Advances in the Astronautical Sciences*, Vol. 116, 2003, pp. 1865–1878; also American Astronautical Society, Paper 03-620.

[6] Storz, M. F., Bowman, B. R., Branson, J. I., Casali, S. J., and Tobiska, W. K., "High Accuracy Satellite Drag Model (HASDM)," *Advances in Space Research*, Vol. 36, No. 12, 2005, pp. 2497–2505. doi:10.1016/j.asr.2004.02.020

[7] Bowman, B., Marcos, F. A., and Kendra, M. J., "A Method for Computing Accurate Atmospheric Density Values from Satellite Drag Data," *Advances in the Astronautical Sciences*, Vol. 119, 2004, pp. 1117–1134; also American Astronautical Society, Paper 04-173.

[8] Bowman, B., "The Semiannual Thermospheric Density Variation from 1970 to 2002 Between 200–1100 km," *Advances in the Astronautical Sciences*, Vol. 119, 2004, pp. 1135–1154; also American Astronautical Society, Paper 04-174.

[9] Cefola, P. J., Proulx, R. J., Nazarenko, A. I., and Yurasov, V. S., "Atmospheric Density Correction Using Two Line Element Sets as the Observation Data," *Advances in the Astronautical Sciences*, Vol. 116, 2003, pp. 1953–1978; also American Astronautical Society, Paper 03-626.

[10] Yurasov, V. S., Nazarenko, A. I., Cefola, P. J., and Alfriend, K. T., "Results and Issues of Atmospheric Density Correction," *Journal of the Astronautical Sciences*, Vol. 52, No. 3, July–Sept. 2004, pp. 281–300.

[11] Yurasov, V. S., Nazarenko, A. I., Alfriend, K. T., and Cefola, P. J., "Reentry Time Prediction Using Atmospheric Density Corrections," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 2, March–April 2008, pp. 282–289.
doi:10.2514/1.26593

[12] Wilkins, M. P., Sabol, C. A., Cefola, P. J., and Alfriend, K. T., "Improving Dynamic Calibration of the Atmosphere," *Advances in the Astronautical Sciences*, Vol. 127, 2007, pp. 1257–1272; also American Astronautical Society, Paper 07-185.

[13] Wilkins, M. P., Sabol, C. A., Cefola, P. J., and Alfriend, K. T., "Validation and Application of Corrections to the NRLMSISE-00 Atmospheric Density Model," *Advances in the Astronautical Sciences*, Vol. 127, 2007, pp. 1285–1304; also American Astronautical Society, Paper 07-189.

[14] Rhoden, E. A., Forbes, J. M., and Marcos, F. A., "The Influence of Geomagnetic and Solar Variability on Lower Thermospheric Density," *Journal of Atmospheric and Solar-Terrestrial Physics*, Vol. 62, 2000, pp. 999–1013.
doi:10.1016/S1364-6826(00)00066-3

[15] Konig, R., and Neumayer, K. H., "Thermospheric Events in CHAMP Precise Orbit Determination," *First CHAMP Mission Results for Gravity, Magnetic and Atmospheric Studies*, edited by C. Reigber, H. Luhr, and P. Schwintzer, Springer, Berlin, 2003, pp. 112–119.

[16] Bruinsma, S., and Biancale, R., "Total Density Retrieval with STAR," *First CHAMP Mission Results for Gravity, Magnetic and Atmospheric Studies*, edited by C. Reigber, H. Luhr, and P. Schwintzer, Springer, Berlin, 2003, pp. 192–199.

[17] Bruinsma, S., and Biancale, R., "Total Densities Derived from Accelerometer Data," *Journal of Spacecraft and Rockets*, Vol. 40, No. 2, March–April 2003, pp. 230–236.
doi:10.2514/2.3937

[18] Bruinsma, S., Tamagnan, S. D., and Biancale, R., "Atmospheric Densities Derived from CHAMP/STAR Accelerometer Observations," *Planetary and Space Science*, Vol. 52, 2004, pp. 297–312.
doi:10.1016/j.pss.2003.11.004

[19] Nerem, R. S., Forbes, J. M., Sutton, E. K., and Bruinsma, S., "Atmospheric Density Measurements Derived from CHAMP/STAR Accelerometer Data," *Advances in the Astronautical Sciences*, Vol. 116, 2003, pp. 1879–1898; also American Astronautical Society, Paper 03-621.

[20] Schlegel, K., Luhr, H., St. Maurice, J. P., Crowley, G., and Hackert, C., "Thermospheric Density Structures over the Polar Regions Observed with CHAMP," *Annales Geophysicae*, Vol. 23, 2005, pp. 1659–1672.
doi:10.5194/angeo-23-1659-2005

[21] Sutton, E. K., Nerem, R. S., and Forbes, J. M., "Global Thermospheric Neutral Density and Wind Response to the Severe 2003 Geomagnetic Storms from CHAMP Accelerometer Data," *Journal of Geophysical Research*, Vol. 110, 2005, Paper A09S40.
doi:10.1029/2004JA010985

[22] Forbes, J. M., Lu, G., Bruinsma, S., Nerem, S., and Zhang, X., "Thermospheric Density Variations due to the 15–24 April 2002 Solar Events from CHAMP/STAR Accelerometer Measurements," *Journal of Geophysical Research*, Vol. 110, 2005, Paper A12S27.
doi:10.1029/2004JA010856

[23] Sutton, E. K., Forbes, J. M., Nerem, R. S., and Woods, T. N., "Neutral Density Response to the Solar Flares of October and November, 2003," *Geophysical Research Letters*, Vol. 33, 2006, Paper L22814.
doi:10.1029/2006GL027583

[24] Bruinsma, S., Forbes, J. M., Nerem, R. S., and Zhang, X., "Thermospheric Density Response to the 20–21 November 2003 Solar and Geomagnetic Storm from CHAMP and GRACE Accelerometer Data," *Journal of Geophysical Research*, Vol. 111, 2006, Paper AO6303.
doi:10.1029/2005JA011284

[25] Bruinsma, S. L., and Forbes, J. M., "Storm-Time Equatorial Density Enhancements Observed by CHAMP and GRACE," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1154–1159.
doi:10.2514/1.28134

[26] Sutton, E. K., Nerem, R. S., and Forbes, J. M., "Density and Winds in the Thermosphere Deduced from Accelerometer Data," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1210–1219.
doi:10.2514/1.28641

[27] Tapley, B. D., Ries, J. C., Bettadpur, S., and Cheng, M., "Neutral Density Measurements for the Gravity Recovery and Climate Experiment Accelerometers," *Journal of Spacecraft and Rockets*, Vol. 44, No. 6, 2007, pp. 1220–1225.
doi:10.2514/1.28843

[28] Doornbos, E., Klinkrad, H., and Visser, P., "Atmospheric Density Calibration Using Satellite Drag Observations," *Advances in Space Research*, Vol. 36, 2005, pp. 515–521.
doi:10.1016/j.asr.2005.02.009

[29] van den IJssel, J., and Visser, P., "Determination of Non-Gravitational Accelerations from G," *Earth Observation with CHAMP Results from Three Years in Orbit*, edited by C. Reigber, H. Luhr, P. Schwintzer, and J. Wickert, Springer, Berlin, 2005, pp. 95–100.

[30] van den IJssel, J., and Visser, P., "Determination of Non-Gravitational Accelerations from GPS Satellite-to-Satellite Tracking of CHAMP," *Advances in Space Research*, Vol. 36, 2005, pp. 418–423.
doi:10.1016/j.asr.2005.01.107

[31] van den IJssel, J., and Visser, P., "Performance of GPS Accelerometry: CHAMP and GRACE," *Advances in Space Research*, Vol. 39, 2007, pp. 1597–1603.
doi:10.1016/j.asr.2006.12.027

[32] Montenbruck, O., van Helleputte, T., Kroes, R., and Gill, E., "Reduced Dynamic Orbit Determination Using GPS Code and Carrier Measurements," *Aerospace Science and Technology*, Vol. 9, 2005, pp. 261–271.
doi:10.1016/j.ast.2005.01.003

[33] McLaughlin, C. A., and Bieber, B. S., Neutral Density Determined from CHAMP Precision Orbits," *Advances in the Astronautical Sciences*, Vol. 129, 2008, pp. 167–186; also American Astronautical Society, Paper 07-260.

[34] McLaughlin, C. A., and Bieber, B. S., "Comparison of Total Density Derived from CHAMP Precision Orbits and CHAMP Acclerometer," *Advances in the Astronautical Sciences*, Vol. 130, 2008, pp. 1193–1206; also American Astronautical Society, Paper 08-177.

[35] Konig, R., Zhu, S., Reigber, C., Neumayer, K. H., Meixner, H., Galas, R., and Baustert, G., "CHAMP Rapid Orbit Determination for GPS Atmospheric Limb Sounding," *Advances in Space Research*, Vol. 30, No. 2, 2002, pp. 289–293.
doi:10.1016/S0273-1177(02)00297-1

[36] Konig, R., Michalak, G., Neumayer, K. H., Zhu, S. Y., Meixner, H., annd Reigber, C., "Recent Developments in CHAMP Orbit Determination at GFZ," *Earth Observation with CHAMP Results from Three Years in Orbit*, edited by C. Reigber, H. Luhr, P. Schwintzer, and J. Wickert, Springer, Berlin, 2005, pp. 65–70.

[37] Michalak, G., Baustert, G., Konig, R., and Reigber, C., "CHAMP Rapid Science Orbit Determination: Status and Future Prospects," *First CHAMP Mission Results for Gravity, Magnetic and Atmospheric Studies*, edited by C. Reigber, H. Luhr, and P. Schwintzer, Springer, Berlin, 2003, pp. 98–103.

[38] Wright, J. R., "Real-Time Estimation of Local Atmospheric Density," *Advances in the Astronautical Sciences*, Vol. 114, 2003, pp. 927–950; also American Astronautical Society, Paper 03-164.

[39] Wright, J. R., and Woodburn, J., "Simultaneous Real-Time Estimation of Atmospheric Density and Ballistic Coefficient," *Advances in the Astronautical Sciences*, Vol. 119, 2004, pp. 1155–1184; also American Astronautical Society, Paper 04-175.

[40] Tanygin, S., and Wright, J. R., "Removal of Arbitrary Discontinuities in Atmospheric Density Modeling," *Advances in the Astronautical Sciences*, Vol. 119, 2004, pp. 1185–1196; also American Astronautical Society Paper 04-176.

[41] Jacchia, J. G., "Revised Static Models for the Thermosphere and Exosphere with Empirical Temperature Profiles," Smithsonian Institution Astrophysical Observatory, Rept. 332, Cambridge, MA, 1971.

[42] Roberts, C. E., Jr., "An Analytic Model for Upper Atmosphere Densities Based upon Jacchia's 1970 Models," *Celestial Mechanics*, Vol. 4, Nos. 3–4, Dec. 1971, pp. 368–377.
doi:10.1007/BF01231398

[43] *COSPAR International Reference Atmosphere*, Akademie-Verlag, Berlin, 1972.

[44] Hedin, A. E., "Extension of the MSIS Thermosphere Model into the Middle and Lower Atmosphere," *Journal of Geophysical Research*, Vol. 96, 1991, pp. 1159–1172. doi:10.1029/90JA02125

[45] Picone, J. M., Hedin, A. E., and Drob, D. P., "NRLMSISE-00 Empirical Model of the Atmosphere: Statistical Comparisons and Scientific Issues," *Journal of Geophysical Research*, Vol. 107, No. A12, 2002, p. 1468. doi:10.1029/2002JA009430

[46] Bowman, B. R., Marcos, F. A., Moe, K., and Moe, M. M., "Determination of Drag Coefficient Values for CHAMP and GRACE Satellites Using Orbit Drag Analysis," *Advances in the Astronautical Sciences*, Vol. 129, 2008, pp. 147–166; also American Astronautical Society, Paper 07-259.

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